

# Mechanical Filters—A Review of Progress

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**Abstract**—This paper is a review of electromechanical bandpass filter, resonator, and transducer development. Filter types discussed include intermediate and low-frequency configurations composed of rod, disk, and flexure-bar resonators and magnetostrictive ferrite and piezoelectric ceramic transducers. The resonators and transducers are analyzed in terms of their dynamic and material characteristics. The paper also includes methods of realizing attenuation poles at real and complex frequencies. The last section is a look at future developments.

## INTRODUCTION

MORE THAN 20 years have passed since the first practical mechanical filters were introduced by Adler [1], Roberts [2], and Doelz [3]. These filters met an existing need for greater selectivity in the IF stages of AM and SSB receivers designed for voice communication. Modern receivers and telephone communication equipments carry not only voice but data messages as well, thus requiring lower passband ripple and well-defined and stable passband limits. In addition, greater selectivity and lower loss are often required.

Mechanical filter development has kept pace with the demands of communication engineers. For instance, filters having passband-ripple requirements of 0.5 dB or less or 60/3-dB bandwidth ratios of 1.3/1 are not unusual. Package size has also been reduced; a 10-pole filter can be designed into a 1-cm<sup>3</sup> package. In addition, the lower frequency limits of mechanical filters are now in the audio range; the upper limit is that of an AT-cut crystal. Although the mechanical filter was conceived in the U.S., a great amount of developmental work is now being conducted in Europe and Japan.

Early interest in the use of mechanical elements to provide passband characteristics resulted from the development of the electrical bandpass filter by Campbell and Wagner in 1917 [4]. A number of patents relating to spring-mass systems [5] were filed soon after, but the most significant work in terms of practical devices was done by Maxfield and Harrison on phonograph recording and playing equipment [6]. Fig. 1 shows a comparison of the frequency response of a phonograph designed as an electromechanical filter (solid curve) and the best of phonographs available at the time. As dramatic as these results were, very little work was done in

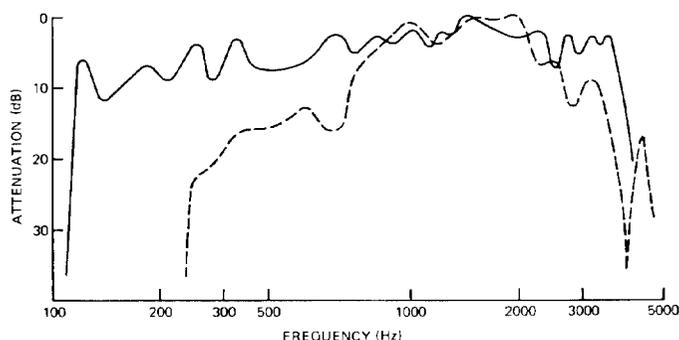


Fig. 1. Improvement in phonograph design through the use of filter design methods [6].

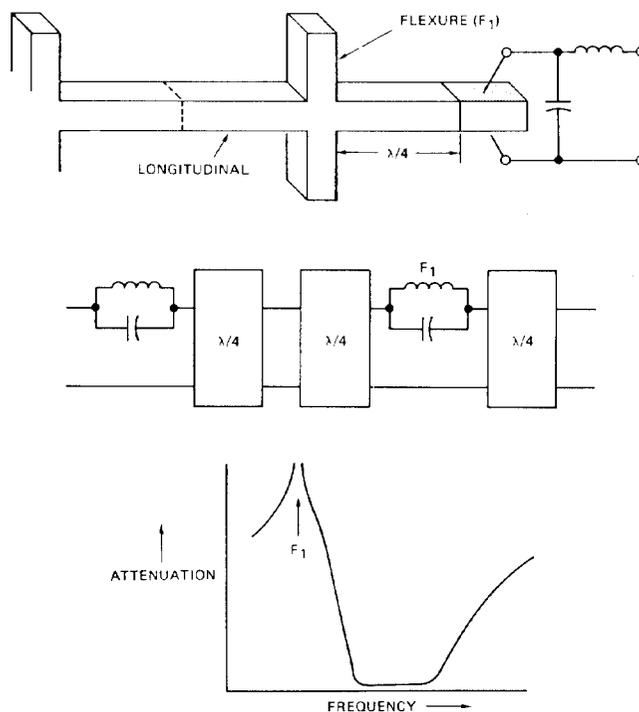


Fig. 2. Distributed element mechanical filter (Mason, 1941).

the next two decades on mechanical filters designed to act as frequency selective devices.

In the early 1940's, Mason followed up his excellent work on crystal filter design with the development of the very interesting, but not widely used, mechanical filter shown in Fig. 2 [7]. Some of the significant features of this design included the use of piezoelectric transducers, distributed-element half-wavelength resonators, and an attenuation-pole-producing coupling element.

Just a few years later, the first practical IF mechanical filter, shown in Fig. 3(a), was developed by Adler.

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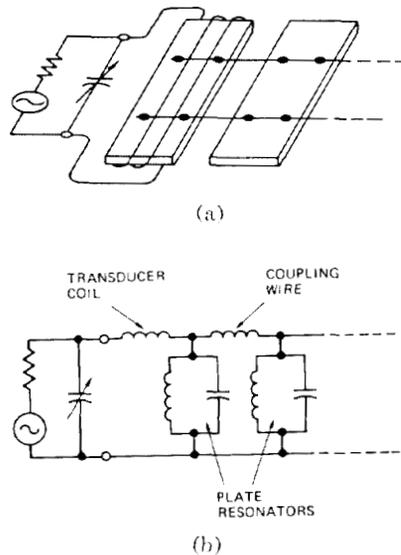


Fig. 3. Resonator-coupling wire mechanical filter (Adler, 1946). (a) Pictorial diagram. (b) Electrical equivalent circuit.

This filter made use of half-wavelength plates coupled by short small-diameter wires. The plate resonators can be represented by parallel tuned circuits, the coupling wires by series inductors, as shown in the electrical equivalent circuit of Fig. 3(b). The nickel end plates act as magnetostrictive input and output transducers.

The frequency response characteristics are monotonic, that is there are no finite attenuation poles. Adler's filter is typical of most modern mechanical filters in that distributed parameter resonators are coupled through nonresonant lines. There is a striking similarity between the plate-wire filter and the latest type of mechanical filter, the monolithic or quartz mechanical filter shown in Fig. 4(a).

The quartz mechanical filter developed by Beaver and Sykes [8] and Nakazawa [9] makes use of the energy-trapping concept where standing waves are set up under each electrode pair. In the regions between the electrodes, the acoustic wave decays exponentially, the electroded regions act like resonators, and the non-electroded regions act like coupling elements. The electrical equivalent circuit of the plate-wire and the quartz mechanical filter shown in Fig. 4(b) are identical except that in the plate-wire case the input and output capacitors are replaced by coils. The monolithic filter also has monotonic frequency response characteristics. A construction similar to that of the monolithic filters described above has been used by Börner and Schüssler [10].

#### BASIC FILTER STRUCTURES

##### Monotonic Designs

Adler's development of the plate-wire mechanical filter was soon followed by Doelz's disk-wire filter [11], shown in Fig. 5(a). The disk-wire filter makes use of flexural modes of vibration of the disk resonators. The two most

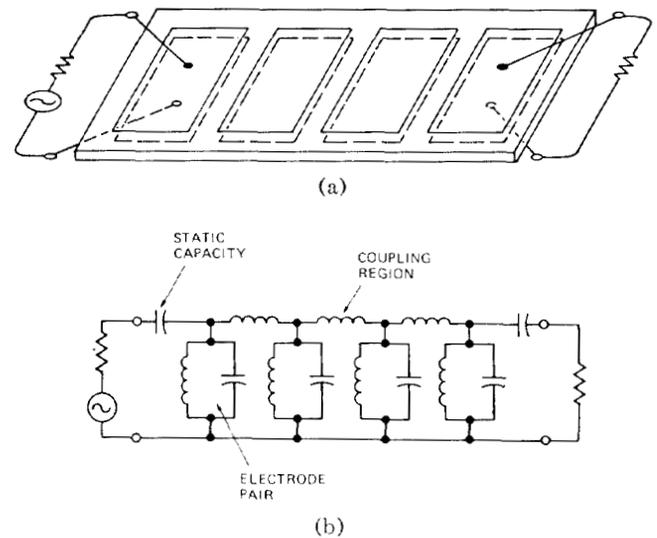


Fig. 4. Quartz monolithic mechanical filter. (a) Pictorial diagram. (b) Electrical equivalent circuit.

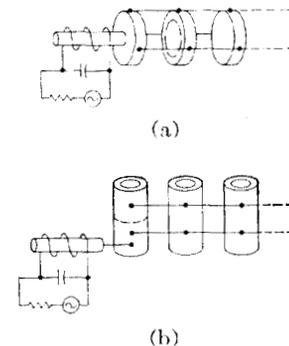


Fig. 5. (a) Disk-wire mechanical filter. (Doelz, 1949). (b) Rod-wire mechanical filter (Tanaka and Börner, 1957).

commonly used modes are the one-nodal circle (50–200 kHz) and the two-circle mode (200–600 kHz). The resonators are coupled by small-diameter wires that are spot welded around the circumference of the disks. As a result of the very complex displacement at the edge of the disk, the coupling mode is a combination of extension and flexure and is, therefore, very difficult to analyze. The earliest designs made use of small-diameter odd-number quarter-wavelength iron-nickel alloy transducers. Later designs, such as that shown in Fig. 5(a), use one-half and full-wavelength ferrite transducers.

Ferrite transducers are also used on the rod-wire filter shown in Fig. 5(b). The torsional rod-wire filter was developed independently by Börner in Germany [12], [13], and Tanaka in Japan [14]; the Tanaka design uses Langevin metal alloy/ceramic transducers in place of the ferrites. The cylindrical rod resonators are designed to vibrate in a half-wavelength torsional mode at frequencies up to 250 kHz and in a half-wavelength longitudinal mode when the filters are designed to operate at 455 kHz. Use of the torsional mode results in longitudinal coupling between resonators, whereas at 455 kHz the coupling involves bending or flexure.

Use of a wire to connect the transducer to the end resonator, as shown in Fig. 5(b), results in a reduction of spurious or unwanted responses, particularly if it is attached at a nodal point of the strongest unwanted modes. This technique, which is often used on disk-wire filters, results in spurious modes being suppressed more than 60 dB, as shown in Fig. 6(a). In contrast to the excellent spurious response rejection of the torsional rod-wire filter is the response [Fig. 6(b)] of the rod-neck filter of Fig. 7(a), which is driven in a similar manner, but is subject to broad bandwidth bending modes.

The rod-neck filter was developed by Roberts [2] at the same time the disk-wire filter was being developed. The basic design concept is that of coupling half-wavelength torsional or longitudinal resonators with quarter-wavelength necks. The two major problems of the early designs were the easily excited spurious bending modes [15], and the difficulty in construction and tuning due to being turned out of a single rod. In addition, at lower frequencies like 100 kHz, the filters become extremely long. In order to reduce the length, Tanaka developed the folded line filter shown in Fig. 7(b). The resonators vibrate in a half-wavelength longitudinal mode and are coupled by relatively large-diameter short wires. The use of longitudinal-mode Langevin transducers plus the reduced overall length results in a greater rejection of unwanted modes. Like the rod-wire and folded designs, the disk-wire filter has a relatively low spurious response level.

All four filters discussed so far operate at frequencies above 50 kHz. With the exception of some work reported by Mason and Konno, there was little activity at frequencies below 50 kHz before 1960. One of the first practical designs was that of Mason and Thurston [16], who used antisymmetric mode flexural resonators coupled in torsion as shown in Fig. 8(a). The antisymmetric mode makes this design less susceptible to microphonic excitation. A more widely used design is the symmetrical mode filter shown in Fig. 8(b). Work by Konno [17], [18], Yakuwa [19], and Albsmeier [20], have made this a very practical device in the frequency range of 300 Hz to 30 kHz. The symmetric-mode filter is driven by piezoelectric-ceramic transducers. The coupling wires are attached to the resonators at the nodal points, which results in torsional coupling. This type of filter is very sensitive to changes in the position of the coupling wires so considerable care is taken in manufacturing to ensure that bending modes are not propagated. The microphonic problem has been solved, in part, by supporting the filter with high-damping silicon-rubber supports.

In addition to the flexural-bar/wire low-frequency mechanical filters, a considerable amount of work has been done in Japan on the development of tuning-fork mechanical filters [21], such as those shown in Fig. 9(a) and (b). The three-prong filter is interesting in that it acts like a coupled two-resonator filter, thus providing a

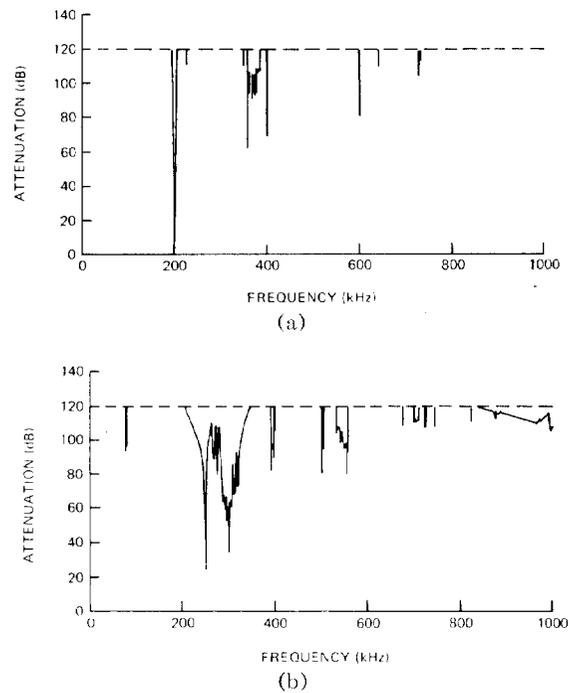


Fig. 6. Spurious responses of (a) torsional rod-wire, and (b) rod-neck mechanical filters.

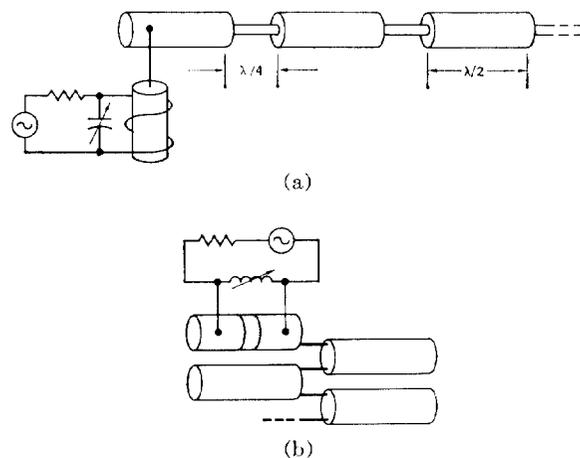


Fig. 7. Distributed element mechanical filters. (a) Rod-neck designs [21]. (b) Folded line designs (Tanaka, 1958).

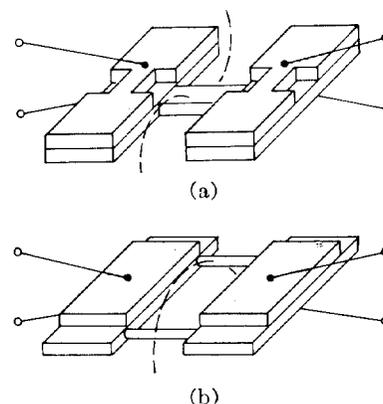


Fig. 8. Low-frequency mechanical filters. (a) Antisymmetric mode [16]. (b) Fundamental mode [18].

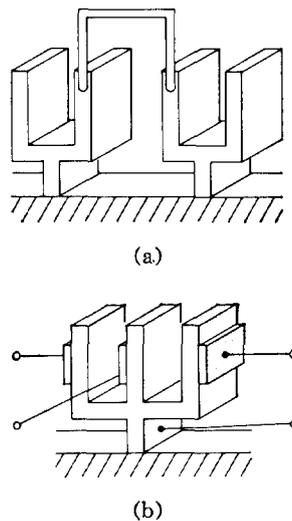


Fig. 9. Tuning fork mechanical filters. (a) U-shaped coupling type. (b) Two-pole three-prong type.

greater amount of selectivity than a simple two-prong device. The tuning fork and flexural-bar filters are widely used in Japan for remote and automatic control, selective calling and paging, telemetry, measuring instruments, and pilot and carrier signal pickup.

#### Finite Attenuation Poles.

By using as many as 15 resonators, highly selective mechanical filters of the bar-wire, disk-wire, and folded-resonator types are realizable. Although the resultant passband amplitude and delay responses are satisfactory for voice communication, the ripple amplitude and differential delay variations may be excessive when data are to be transmitted. By making use of finite-frequency attenuation poles, fewer resonators are needed, thereby reducing the differential delay, size, cost, and, most often, the passband ripple.

One of the first practical devices to realize finite attenuation poles was the crystal-plate filter with capacitive bridging [22], [23], shown in Fig. 10(a). Although the plate configuration was described in Adler's basic patent [24], the idea of using quartz and capacitive bridging was new. Connecting the capacitor from the top electrode of the input plate to the bottom electrode of the output plate is the same as adding a phase inverter across the output terminals of the equivalent circuit of Fig. 10(b) and results in a pair of attenuation poles as shown in Fig. 10(c). The phase inverter adjacent to the coupling element represents the 180-deg phase shift between resonators at the lowest natural mode of the mechanical system. The resonators vibrate in a length-extension mode ( $5^\circ - X$  or  $-18.5^\circ - X$ ) or face shear mode (CT) and are coupled through so-called Poisson coupling. Although these filters have been designed and built with more than two resonators, most currently manufactured filters are designed as 455-kHz 2 poles in cascade for use in SSB and FM receivers.

The use of wires to couple mechanically alternate

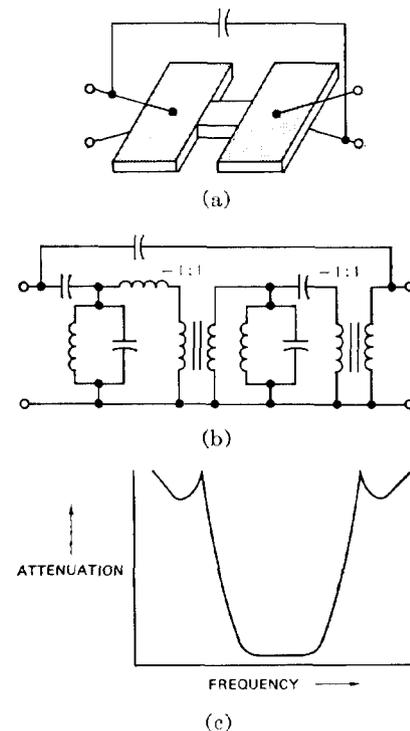


Fig. 10. Crystal mechanical filter with capacitive bridging [22]. (a) Pictorial diagram. (b) Electrical equivalent circuit. (c) Frequency response.

resonators was independently considered by Börner [25] and Johnson [26]. Börner's filter shown in Fig. 11(a) makes use of half-wavelength torsional-mode resonators and extensional mode-coupling elements. The disk-wire filter of Fig. 11(b) is composed of flexural mode disks and basically extensional mode-coupling elements. When the spacing between the resonators is such that the bridging wire is less than one-half of an acoustic wavelength long, the ideal transformer of Fig. 11(c) acts like a simple one-to-one transformer and an attenuation pole is realized on the high-frequency side of the filter passband. This can be understood by converting the  $\pi$  network, composed of the two adjacent resonator coupling wires (inductors) and the bridging wire, to its  $T$  equivalent circuit. The inductor in series with the center resonator produces an impedance zero, which results in an attenuation pole above the filter passband, as shown in Fig. 11(d).

When the bridging wire is between one-half and a full-wavelength long, the transformer in the electrical equivalent circuit acts like a phase inverter and an attenuation pole is realized below the filter passband. An alternate method of realizing the phase inversion in the rod-wire filter is shown in Fig. 11(a) by the dashed lines. In this case, out-of-phase regions of the alternate resonators are coupled. A similar method is used with disk-wire filters where one of the alternate disks vibrates in a diameter mode (rather than a nodal-circle mode), the three adjacent disk wires being connected to in-phase sectors, and the bridging wire to an out-of-phase portion of the resonator [27].

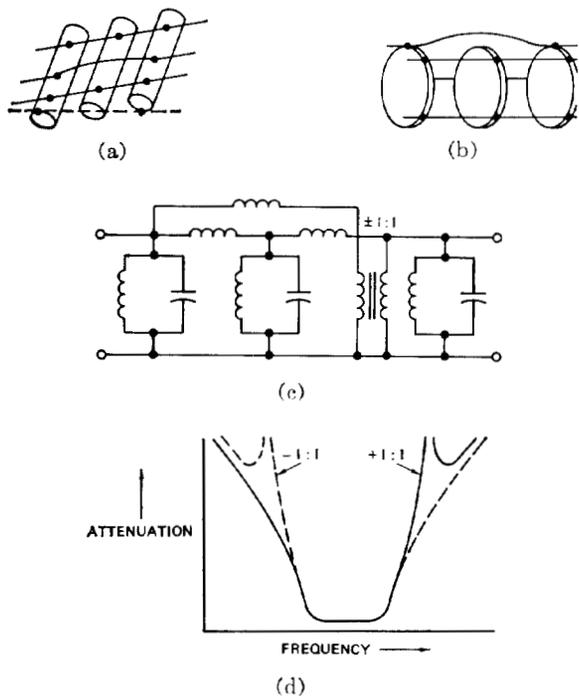


Fig. 11. Single-resonator acoustic bridging. (a) Rod-wire, [25]. (b) Disk-wire (Johnson, 1964). (c) Electrical equivalent circuit. (d) Frequency response.

If a high degree of selectivity is needed both above and below the filter passband, a coupling wire can be used to bridge two resonators producing a symmetrical pair of attenuation poles. This type of design is subject to being able to realize a phase inverter with each bridging wire. When no phase inversion takes place, the resultant frequency response is less selective than the monotonic case but delay compensation due to right-half plane attenuation poles is possible (see Table I). The most common method used to realize the phase inverter, in the case of the rod-wire filter, is to space adjacent resonators a quarter-wavelength apart, which results in the bridging wire being three-quarters of a wavelength long. The advantages of the quarter-wavelength coupling are that the resonators are all tuned to the center frequency of the filter and, in addition, the coupling is relatively insensitive to variations in wire length.

In order to maintain a small package size, the coupling wire length between disk resonators is usually less than one-eighth of a wavelength. At 455 kHz, due to the thickness of each disk being on the order of a quarter-wavelength, simple two-disk bridging results in symmetrical finite-attenuation poles. Fig. 12 shows the frequency response of the 12-disk filter shown in Fig. 13. Note that the passband ripple is quite low. This is in part due to the fact that the disk-wire as well as the rod-wire finite-pole filters are designed with modern insertion-loss techniques. Included in the design method are transformations such as that shown in Fig. 14(a) where a low-pass or bandpass ladder network is converted to an equivalent bridged form [28]. Because of the narrow-band nature of mechanical filters, an inverter  $I$  can be

TABLE I  
ATTENUATION POLE LOCATIONS FOR VARIOUS  
BRIDGING CONFIGURATIONS

	Direct (1 : 1)	Inverted (-1 : 1)
One resonator	Upper stopband pole	Lower stopband pole
Two resonators	Delay correction	Upper and lower stopband poles

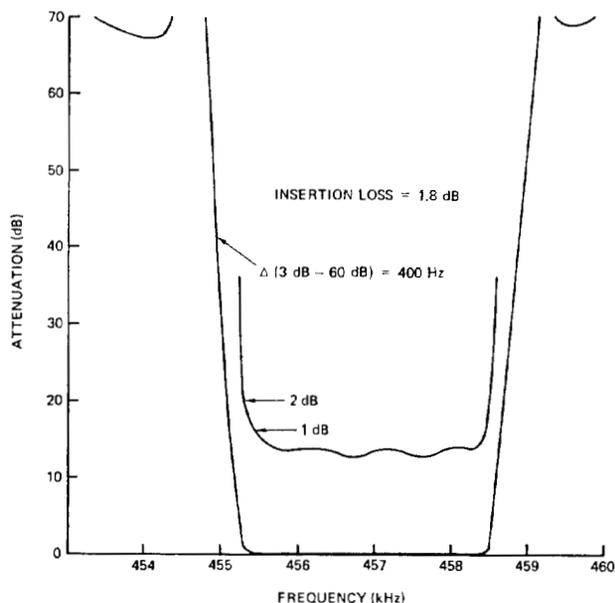


Fig. 12. Frequency response of a double-disk bridging mechanical filter.

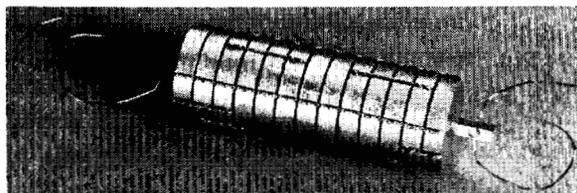


Fig. 13. Mechanical filter with wire bridging across two resonators (Collins Radio).

converted to a  $\pi$  network composed of two negative inductors and one positive inductor, all having the same absolute value [29]. As an example, the low-pass network of Fig. 14(b), which may be a Cauer-type filter [30], can be converted to the bandpass double-resonator bridging topology shown in Fig. 14(c). The use of double (coincident) poles results in a physically symmetrical mechanical configuration, which is helpful in reducing manufacturing costs. The short coupling wire length between resonators, as shown in Fig. 13, not only decreases the package size but increases the strength of the filter. The ability of the structure to withstand high shock and vibration levels is also due to the coupling wires being located away from the centroid of the structure.

For the past 15 years, disk-wire mechanical filters

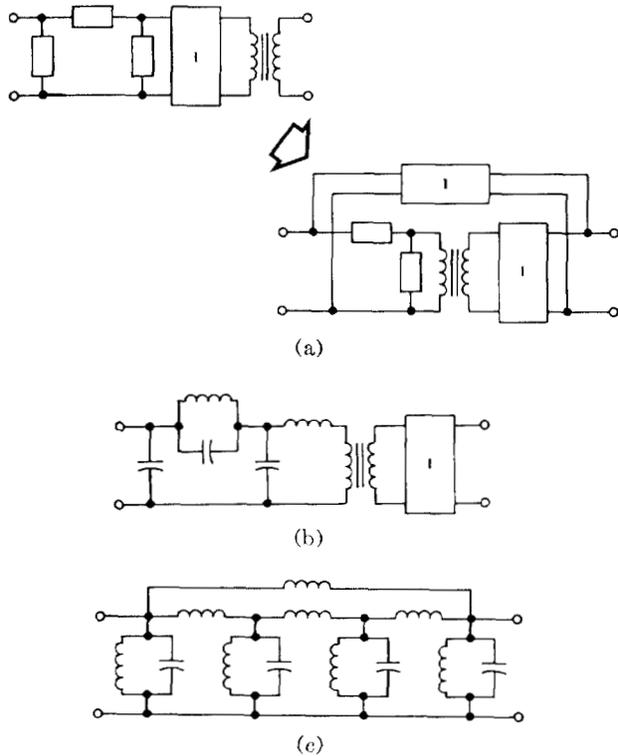


Fig. 14. Double bridging transformations. (a) General transformation. (b) Low-pass prototype. (c) Final bridging-wire bandpass electrical equivalent circuit.

have been designed to make use of spurious modes of vibration to control stopband selectivity. Although the theory was not understood until recently [31], it was found that by varying the coupling-wire orientation around the circumference of the disks the slope of one side of the response could be increased at the expense of the other side. For instance, in Fig. 15(a) each disk has a natural resonance near-frequency  $F_1$  in the passband. Each disk also has a natural resonance (actually two as well as many others at different frequencies) near  $F_2$  above the filter passband. Making use of the simplified equivalent circuit shown in Fig. 15(b), we see that an attenuation pole is produced at  $F_\infty$  between  $F_1$  and  $F_2$ . This results in a steeper response above the filter passband, which can be controlled by varying the coupling wire orientation, which in turn controls the effect of resonator  $F_2$ . The same technique has been used in the design of plate filters [32] where the length and width dimensions control the frequencies  $F_1$  and  $F_2$  as shown in Fig. 15(c).

By removing a segment from the edge of a disk resonator, two controllable diameter modes corresponding to each pair of degenerate modes can be produced [33]. This technique has been used to design a variety of multiple mode filters [34]–[36], most of which are still only laboratory models. An example of this type of design is shown in Fig. 16(a). This particular filter employs one-circle one-diameter modes of vibration, as well as piezoelectric ceramic transducers and has a fre-

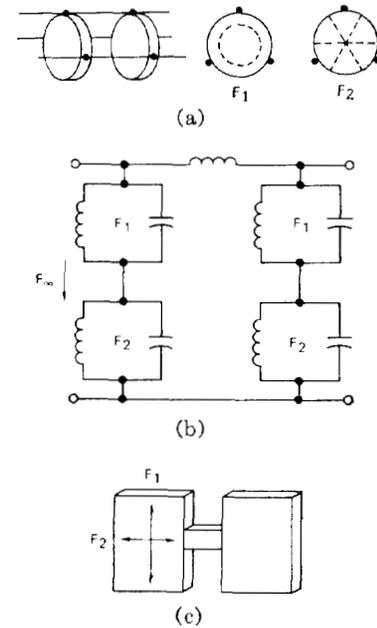


Fig. 15. Multiple-mode finite pole configuration. (a) Disk-wire. (b) Simplified electrical equivalent circuit. (c) Plate-type design.

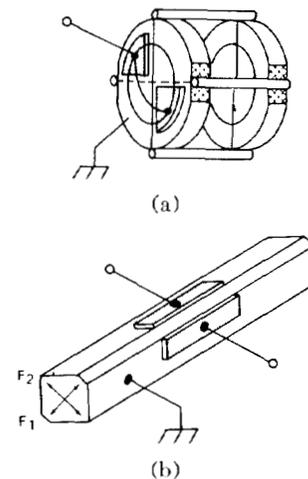


Fig. 16. Multiple resonant mode filters. (a) Disk-wire design. (b) Low-frequency flexural bar.

quency response equivalent to a four-resonator design. The solid nodal lines correspond to the highest natural mode.

In the case of low-frequency filters, a similar technique can be used where the corner of a flexural bar can be removed to produce two natural modes by destroying symmetry of the moments of inertia. An example of this type of resonator is shown in Fig. 16(b) where the arrows show the displacement directions of the two modes [37]. Fig. 17 shows an early filter that makes use of this technique to obtain a total of six natural resonances as well as additional attenuation poles [38]. A large variety of devices have been designed using this method, including a three-resonance two-attenuation pole filter constructed from a single bar [39].

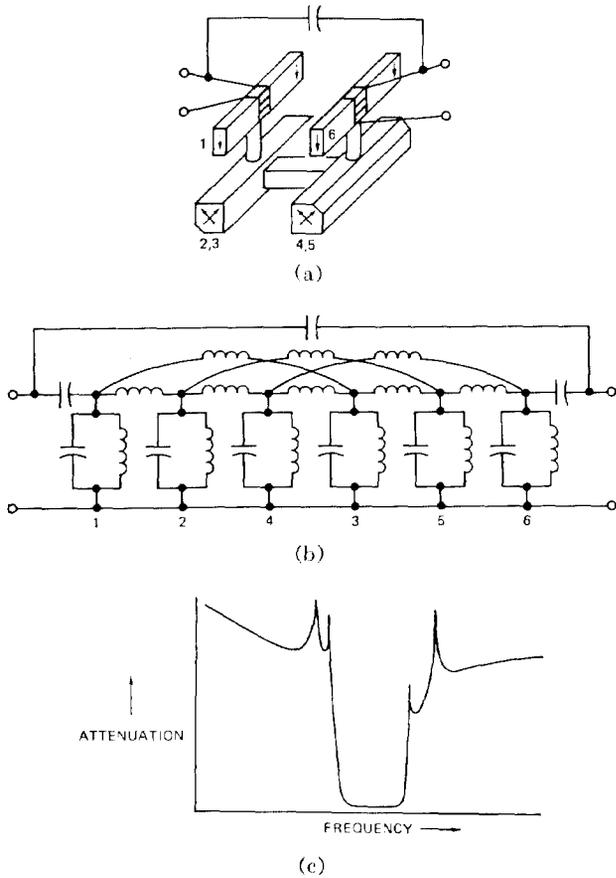


Fig. 17. Six-pole-segmented flexural bar design.

ANALYSIS OF RESONATORS AND COUPLING ELEMENTS

A considerable amount of work has been done in the past few years on the problem of analyzing and modeling mechanical resonators and coupling elements. This work has resulted in a better understanding of the effects of unwanted modes found in both resonators and coupling wires on mechanical filter response characteristics. In addition, the improved network models have been most helpful in the development of new mechanical filter types.

Lumped Element Equivalent Circuits

The driving point mobility (ratio of velocity to force) of a linear mechanical resonator at a point  $i$  can be expressed as [40]

$$v_i/f_i = (\omega/j) \sum_{j=1}^{\infty} [M_{ij}(\omega^2 - \omega_j^2)]^{-1} \quad (1)$$

where  $v_i$  and  $f_i$  are the velocity and the applied force at  $i$  and  $M_{ij}$  is the equivalent mass of mode  $j$  corresponding to the natural frequency  $\omega_j$ . By making use of a velocity transformer of turns ratio  $\phi_{ij}$  and recognizing (1) as the partial fraction expansion of a reactance function, we can construct the schematic diagram shown in Fig. 18(a).

This network describes the characteristics of a two-mode resonator at a point (or port in electrical termi-

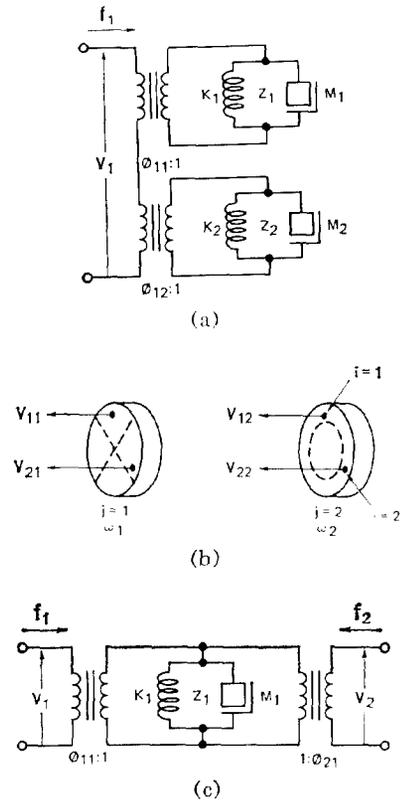


Fig. 18. Two-port two-mode disk resonator. (a) Driving point impedance model. (b) Disk nodal patterns. (c) Single-mode two-port equivalent circuit.

nology)  $i = 1$ . The resonator could be a rod or bar vibrating in a flexural and a longitudinal mode or, as shown in Fig. 18(b), a disk vibrating in a two-diameter flexure mode and a single-circle flexure mode. If only one of the modes, but two points on the disk are considered, the force-velocity relationships can be found from the schematic diagram of Fig. 18(c). In general, a resonator having  $M$  ports and  $N$  natural modes can be described by the matrix equation

$$v] = [Z] f] \quad z_{kl} = \sum_{j=1}^N \phi_{kj} \phi_{lj} z_j \quad (2)$$

In (2),  $v]$  and  $f]$  are column matrices of order  $M$  and  $z_{kl}$  is an element in the  $M \times M$   $Z$  matrix. A generalized equivalent circuit based on (2) is shown in Fig. 19.

Disk Resonators

To be able to make use of any resonator model, it is necessary to be able to calculate or measure natural resonant frequencies and equivalent mass values at specific points on the resonator. In the case of thick disk resonators, excellent agreement between calculated and experimental values of frequency were obtained by Deresiewicz and Mindlin [41] and Sharma [42] for symmetrical modes; that is, flexural modes having circular nodal patterns. More recently, Onoe solved the problem of nonaxisymmetrical modes (nodal diameters)

by making use of a linear combination of independent waves that satisfy the differential equations of motion and boundary conditions on the major surfaces of the disk as well as approximately satisfying boundary conditions on the lateral surfaces [43]. Unlike the exact solutions that are expected when analyzing an electrical network, solutions for frequency and equivalent mass of a mechanical resonator are only as good as the number of higher order waves that are taken into account. The greater the number of waves, the more accurately the boundary conditions can be satisfied.

The earliest work on finding the equivalent mass of a thick disk resonator was performed by Sharma for the case of axisymmetric modes [44]. This analysis was based on the method of finding the equivalent mass by dividing the total kinetic energy in the system (disk) by one-half of the square of the velocity in a specified direction at a point on the disk [45]. This same technique coupled with that of Onoe's [43] has been used to calculate the equivalent mass of disks vibrating in both symmetric and nonsymmetric modes [46]. Examples of equivalent mass versus position on the surface of the disk are shown in Fig. 20 for two adjacent vibration modes. Note that the equivalent mass at the disk edge ( $r = 1.0$ ) is lower in the case of the adjacent two-diameter one-circle mode than the two-circle mode. As a rule, the lower the impedance of a mode, the broader is its coupled response and the more difficult it is to suppress.

Within the frequency range of interest (50–600 kHz) in the case of disk-wire filters, there are various other unwanted modes present such as radial and concentric shear modes. These can often be troublesome in the case of wideband filter designs where the modes sometimes fall in the range of the passband. These particular modes, called contour modes because they involve no transverse (flexural) vibration but only a change in the shape or contour of a disk, have studied by Onoe [47].

*Bar Resonators and Coupling Elements*

Flexural-mode resonators are also used at low frequencies. In the 500-Hz-to-50-kHz frequency range, the resonators are in the form of bars such as those shown in Fig. 8. Analysis of the resonant frequencies of thick flexural-bar resonators has been performed by Mason [45] and Näser [48]. If the resonator is treated as a thin bar, equations similar to those describing a transmission line can be written as [15], [49]

$$\begin{bmatrix} F_1 \\ M_1 \\ V_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} H_0 & -H_8(\alpha/l) \\ H_7(\alpha/l) & H_6 \\ H_8(j\omega l^3/K\alpha^3) & H_{10}(j\omega l^2/K\alpha^2) \\ -H_{10}(j\omega l^2/K\alpha^2) & H_7(j\omega l/K\alpha) \end{bmatrix}$$

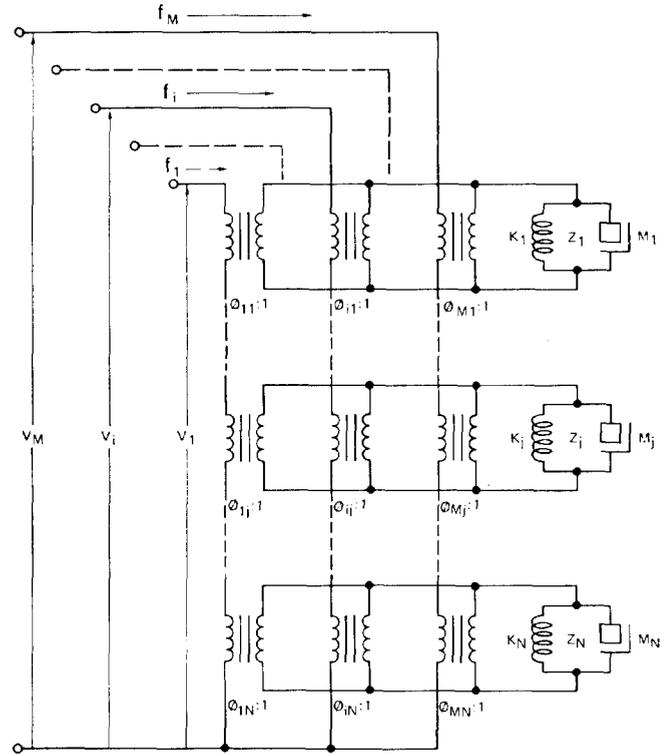


Fig. 19. Generalized schematic of an  $M$ -port  $N$ -node resonator.

$$\begin{aligned} H_1 &= S \cdot s & H_6 &= S \cdot c + C \cdot s & S &= \sinh \alpha \\ H_2 &= C \cdot c & H_7 &= s + S & s &= \sin \alpha \\ H_3 &= C \cdot c - 1 & H_8 &= s - S & C &= \cosh \alpha \\ H_4 &= C \cdot c + 1 & H_9 &= c + C & c &= \cos \alpha \\ H_5 &= S \cdot c - C \cdot s & H_{10} &= c - C & & \\ \alpha^4 &= (\rho A/K)\omega^2 l^4 & K\alpha^2/(j\omega l^2) &= (\rho AK)^{1/2}/j. & & \end{aligned}$$

In (3), which relates to Fig. 21(a),  $K$  represents the product of Young's modulus times the moment of inertia of the cross section of the bar, and  $\rho$ ,  $A$ , and  $l$  are, respectively, the density, cross sectional area, and length of the bar.

Using (3), we can represent a flexural-mode resonator by the equivalent circuit shown in Fig. 21. This equivalent circuit represents a one-resonator two-port system similar to that shown in Fig. 18(c). Note that in this case the representation (where force is an across variable) is used resulting in a dual formulation where the resonator is represented by a series-tuned circuit. In addition-

$$\begin{bmatrix} -H_7(K\alpha^3/j\omega l^3) & -H_{10}(K\alpha^2/j\omega l^2) \\ H_{10}(K\alpha^2/j\omega l^2) & -H_8(K\alpha/j\omega l) \\ H_9 & -H_7(l/\alpha) \\ H_8(\alpha/l) & H_9 \end{bmatrix} \begin{bmatrix} F_2 \\ M_2 \\ V_2 \\ \theta_2 \end{bmatrix} \quad (3)$$

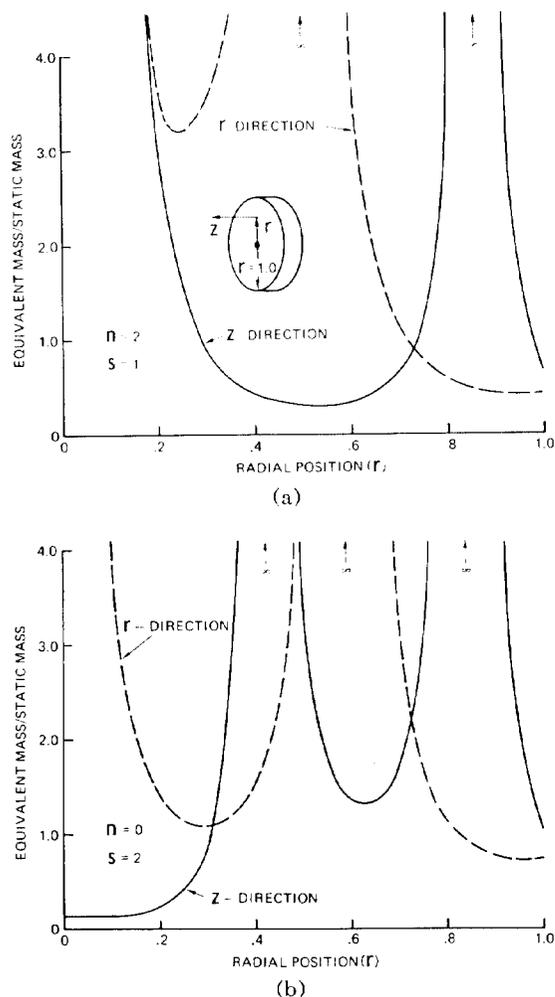


Fig. 20. Disk resonator equivalent mass in the radial  $r$  and axial  $z$  directions for the (a) 2-nodal diameters and 1-nodal circle, and (b) 2-nodal circle cases.

tion, the two ports or points on the resonator are actually represented in the equivalent circuit by four ports, two for linear motion and two for rotation. As in the earlier disk case, a general equivalent circuit can be drawn without difficulty by parallel connecting networks of the form shown in Fig. 21(b) [49]. Konno's normalized function  $\Xi$  is similar to the  $\phi$  function of (2) when the impressed bending moments are equal to zero.

The equivalent circuits of Fig. 21(b) and (3) are very useful in the case of a rod-wire filter where the rod resonator vibrates in an extensional mode [50]. Fig. 22(a) shows a coupling wire attached to the ends of two resonators. The coupling wire is driven in flexure, but there is no rotation of the wire at the points of contact with the resonator, i.e.,  $\theta_1 = \theta_2 = 0$ . After some manipulation of (3), we can write

$$\begin{bmatrix} V_1 \\ F_1 \end{bmatrix} = \begin{bmatrix} H_6/H_7 \\ (1/j)(\alpha^3 K/\omega l^3)(2H_1/H_7) \end{bmatrix}$$

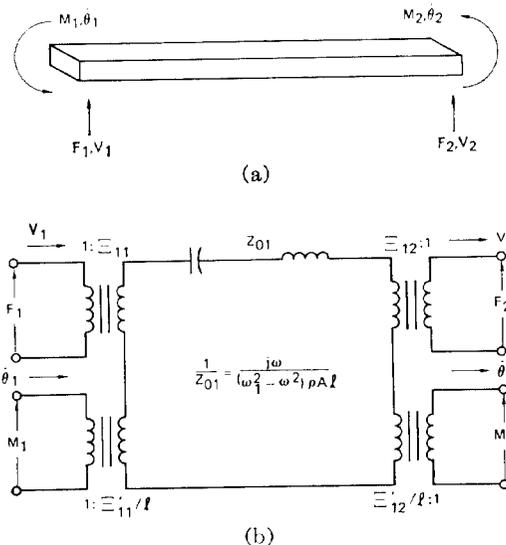


Fig. 21. Flexure mode bar. (a) Pictorial diagram. (b) Classical analogy (force-voltage) equivalent circuit representation.

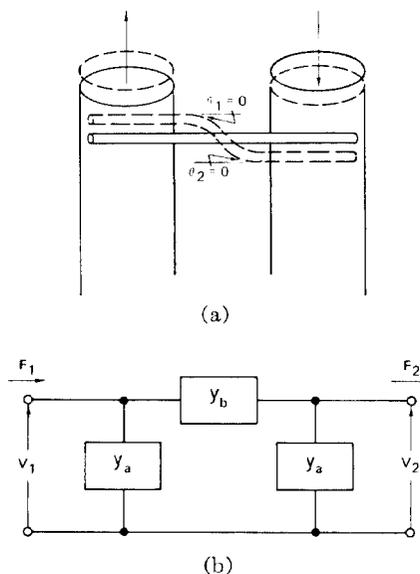


Fig. 22. (a) Longitudinal bar-resonator wire-coupling element (case of zero rotation). (b)  $\pi$  equivalent circuit.

The  $ABCD$  matrix of (4) can be transformed to a  $y$  matrix, which, in turn, can be used to calculate  $y_a$  and  $y_b$  of the  $\pi$ -equivalent circuit of Fig. 22(b). We find that

$$\begin{aligned} Y_a &= j(K\alpha^3/\omega l^3)[(H_6 + H_7)/H_3] \\ Y_b &= -j(K\alpha^3/\omega l^3)(H_7/H_3). \end{aligned} \tag{5}$$

The coupling wire acts as a quarter-wavelength line when  $H_6 = 0$ . In this case, we see from (5) that  $y_a = -y_b$

$$j(\omega l^3/\alpha^3 K)H_3/H_7 \begin{bmatrix} V_2 \\ F_2 \end{bmatrix} \tag{4}$$

and from (4) after some manipulation that [50]

$$\begin{bmatrix} V_1 \\ F_1 \end{bmatrix} = \begin{bmatrix} 0 & j(\omega l^3 / \sqrt{2} \alpha^3 K) \\ (1/j)(\sqrt{2} \alpha^3 K / \omega l^3) & 0 \end{bmatrix} \begin{bmatrix} V_2 \\ F_2 \end{bmatrix}. \quad (6)$$

An analysis of an extensional-mode resonator or coupling wire is somewhat less difficult than that of a flexural element. A rod or wire vibrating in an extensional mode acts as a simple transmission line and, thus, can be described by the *ABCD* matrix

$$\begin{bmatrix} V \\ F_1 \end{bmatrix} = \begin{bmatrix} \cos(\beta l) & jZ_0 \sin(\beta l) \\ j(\sin(\beta l)/Z_0) & \cos(\beta l) \end{bmatrix} \begin{bmatrix} V_2 \\ F_2 \end{bmatrix} \quad (7)$$

where  $\beta = \omega \sqrt{\rho/E}$  and  $Z_0 = (A \sqrt{\rho E})$ .  $E$  is Young's modulus. Extensional-mode resonators and coupling elements have been described in considerable detail in [14] and [40].

### Resonator Materials

Because of transducer bandwidth limitations, the presence of unwanted modes of vibration and competition with *LC* and ceramic filters, the ratio of bandwidth to center frequency in the case of mechanical filters is usually less than 10 percent but more commonly about 1 percent. This small fractional bandwidth requires the resonators to have a temperature coefficient of frequency of 1–10 ppm/°C, a corresponding low aging rate, and a  $Q$  value of at least 10 000.

The major contribution to the variation of frequency in a metallic alloy with temperature is the change in the stiffness or Young's modulus of the material. Iron-nickel alloys that contain either 27 or 44 percent Ni have a low-temperature coefficient of stiffness but are relatively unstable with regard to changes of the percentage of Ni. By adding chromium, the stability can be improved considerably, but the aging characteristics and  $Q$  values of these so-called Elinvar materials are not acceptable. The addition of titanium or beryllium to the Fe–Ni–Cr improves both aging and  $Q$  and, in addition, makes it possible to vary the temperature coefficient of resonant frequency material by cold work and heat treating [51].

In order to obtain a low temperature coefficient, the Fe–Ni–Cr and Be or Ti material is first solution annealed, then quenched, and then cold worked 15–50 percent. Next, the material is precipitation hardened (Ni is precipitated from a supersaturated solution by the Be or Ti) by heat treatment at 400°–675°C for at least two hours. The amount of cold work and temperature time determines the temperature coefficient of the material. Fig. 23(a) and (b) show temperature curves of Thermanalast 5409 (Be) and Ni-Span C (Ti) after having been adjusted for best temperature coefficient. Both materials show frequency shifts of less than 50 Hz over a temperature range of 100°C at 500 kHz.

The addition of Be or Ti has the effect of reducing the aging rate to less than  $1 \times 10^{-7}$  ppm/week or approximately 25 Hz at 500 kHz over a period of 10 years. The

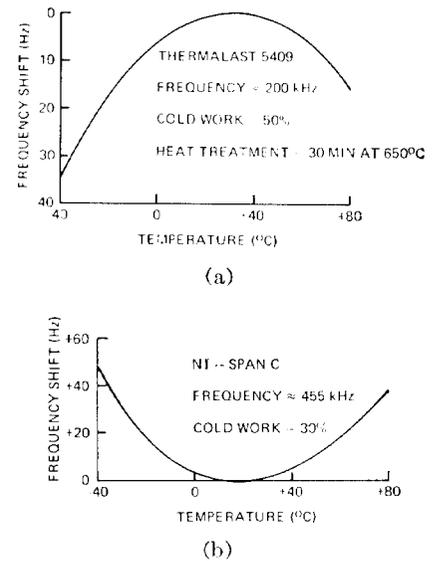


Fig. 23. Constant-modulus resonator characteristics. (a) Thermanalast 5409 torsional rod resonator. (b) Ni-Span C flexure mode disk.

Be or Ti also improves the  $Q$  of the resonators, which may vary from 10 000 to 25 000, depending on the amount of cold working and precipitation hardening. A value of 20 000 is typical for most applications and causes only a small amount of loss at the passband edges of a 1 percent fractional bandwidth filter. For instance, the response of a 3-kHz bandwidth filter at 200 kHz is practically that of a lossless network.

When adjusted for the best temperature coefficient, the resonator frequency shifts have little effect on the passband ripple. The passband-ripple variation will, for the most part, be determined by the characteristics of the transducer.

## TRANSDUCERS AND TRANSDUCER MATERIALS

### Transducer Configurations

The most widely used transducers have been the simple magnetostrictive ferrite rod transducer (Fig. 5) and the Langevin ceramic-metal alloy transducer [Fig. 7(b)] for frequencies above 60 kHz and the composite or sandwich-type of ceramic-metal transducer at low frequencies (Fig. 8). Although these are the most popular, filters are being manufactured that use longitudinal mode ceramic rods, iron-nickel alloy wires, and various quartz crystal cuts.

An electromechanical transducer, regardless of the type, can be characterized by its resonant frequency, coupling coefficient, and static reactance. As an example, the magnetostrictive ferrite transducer shown in Fig. 24 can be defined by the mechanical resonant frequency  $f_1$  (actually the mechanical resonance with the electrical terminals open circuited), the electromechanical coupling coefficient, which relates the electrical and mechanical parameters, and the inductance of the transducer coil  $L_1$ . If the transducer is directly attached to the end reso-

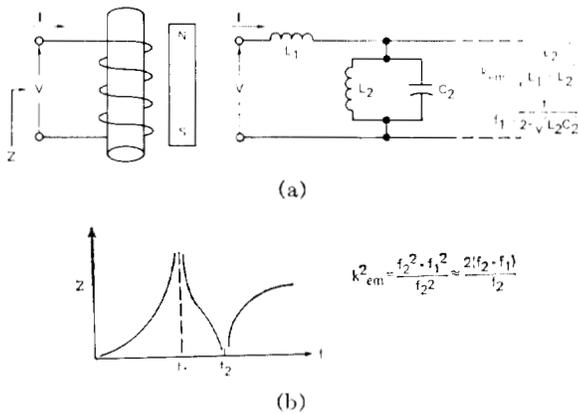


Fig. 24. Magnetostrictive transducer characteristics.

nator, it affects the resonant frequency of the combination in direct proportion to its relative equivalent mass. In the case of the electromechanical coupling coefficient, it is important that it be constant because it directly determines the transformation of the electrical terminating resistance into a mechanical termination that must be matched to the mechanical impedance of the filter. In most applications, the electrical inductance is resonated with a capacitor, which is used for temperature compensation in critical cases.

The filter designer has the task of finding the most stable transducer that will, in addition, properly match the electrical and mechanical networks. This involves choosing a material with a stable coupling coefficient first and then a configuration that reduces the transducer equivalent mass to a minimum so as to reduce the frequency shift with temperature. This can be done by decreasing the diameter of the ferrite rod or the thickness of the ceramic in a Langevin transducer. The size reduction is, in turn, limited by the sensitivity of the coil or static capacity, or in the case of wide-bandwidth filters by the fact that, in the limit, the electrical inductor-capacitor combinations actually become the end resonators.

*Magnetostrictive Ferrite Transducers*

The most widely used filter transducer material is a cobalt-substituted iron-nickel ferrite, which was specially developed for use in electromechanical filters [52]. Addition of 0.6 percent cobalt results in a highly stable coupling coefficient, whereas 1.0 percent cobalt results in a very small variation of frequency  $f_1$  with temperature. Fig. 25 shows the variation of coupling coefficient and frequency  $f_2$  with change in temperature for a practical design at 200 kHz [53]. In order to be able to reduce the effect of the transducer on the end resonator frequency, optimum values of permanent magnet bias and coil dimensions must be chosen so as to maximize the coupling coefficient. The coupling coefficient of 10 percent shown in Fig. 25, which is typical for a well-designed transducer, is also the limiting value of the filter fractional bandwidth. The coupling coefficient varia-

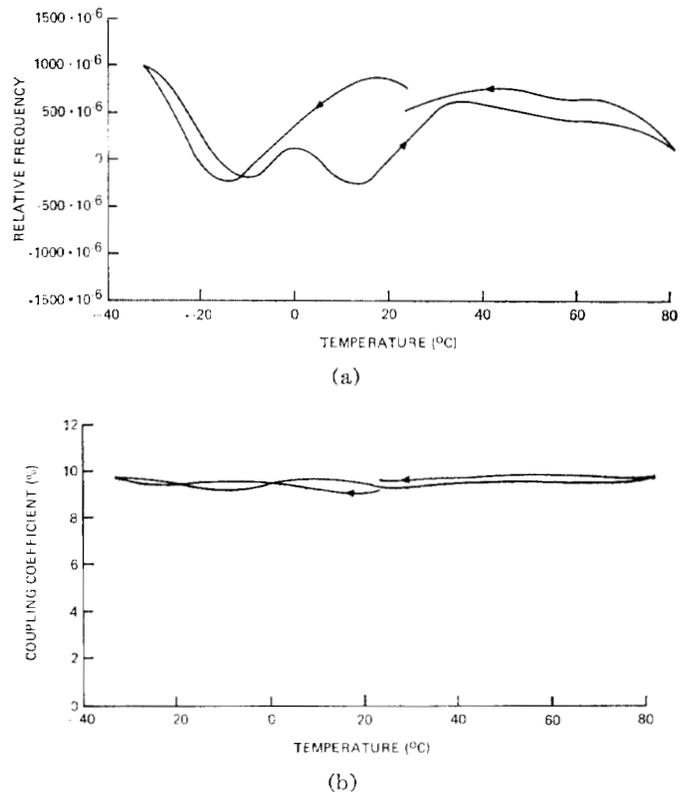


Fig. 25. Characteristics of Telefunken S3 ferrite transducers. (a) Frequency variation. (b) Coupling coefficient variation with change in temperature (longitudinal mode).

tion is approximately 5 percent, which is equivalent to a 10 percent change in the terminating resistance, which may or may not be excessive depending on the application. Variations of resonant frequency with temperature are on the order of 0.1 percent or approximately 500 Hz at 500 kHz over a temperature range of 100°C. Transducer coil  $Q$  varies from 10 to 40 or better, depending on whether a metal alloy or a ferrite shield is used.

Some of the advantages of using magnetostrictive ferrite as a transducer material as opposed to a piezoelectric ceramic, for example, are ease of manufacturing a material with consistent properties, excellent stability of coupling and frequency with time as the result of using a stable permanent magnet bias, and simple capacitive tuning. In the case of high-performance channel filters, excellent matching between the filter and the external circuit can be achieved by the use of a series-parallel combination of tuning (and matching) capacitors. In the case of high-performance piezoelectric ceramic-transducer mechanical filters, transformers are used for both tuning and impedance matching, whereas in the case of inexpensive miniature filters, no tuning is necessary [54].

*Piezoelectric Ceramic Transducers*

Whereas in the U. S. and Germany, the most early transducer development was centered around magnetostrictive ferrite transducers, in Japan most initial work, in fact most current work, involves piezoelectric ceramic

transducers. The barium-titanate material used in early designs suffered from large changes in coupling coefficient and frequency with temperature, which, in turn, resulted in large passband-ripple variations with temperature. The development of lead-zirconate-titanate (PZT) materials [55], which show vastly improved temperature characteristics, has made it possible for piezoelectric ceramic materials not only to compete with, but in a number of cases to replace, magnetostrictive ferrite transducers.

The principal advantage of the piezoelectric ceramic is its excellent electromechanical coupling coefficient. Whereas a bar-type ferrite transducer has a coupling coefficient of 10 percent, a side-plated bar of temperature-stable piezoelectric ceramic has a coupling coefficient between 15 and 25 percent. The higher coupling coefficient therefore makes it possible to use a minimum amount of ceramic material as compared to a highly temperature-stable metal disk alloy in the design of composite end resonator/transducer assemblies. In addition, the higher electrical  $Q$  of the piezoelectric material results in lower filter insertion loss and its greater inherent linearity results in lower intermodulation distortion. Fig. 26 shows the variation of frequency and coupling with temperature for the case of a PZT ceramic that has a coupling coefficient  $k_{13}$  of approximately 23 percent.

The most widely used of the ceramic transducers is the Langevin type, which is in the form of a rod composed of a ceramic disk sandwiched between two metal alloy rods, Fig. 7(b). The high planar coupling coefficient of a thin disk, which is on the order of 25–40 percent (for stable materials), and the Langevin configuration make it possible to design resonator/transducer assemblies that have high mechanical  $Q$  and good temperature characteristics with only a small reduction in the coupling coefficient [54]. Development of this type of transducer for filter applications was started approximately 15 years ago by Tanaka [14] and Tagawa [56] in Japan. Because of the instability and low  $Q$  of most bonding materials such as epoxy, coupled with the fact that the bond has an appreciable thickness, the ceramic/metal alloy attachment problem is one of importance and has required a great deal of effort in its solution.

In the past few years, there has been a considerable amount of analytical work done in Germany on Langevin-type transducers [54], [57], and [58]. This work includes equations that describe the variation of coupling, frequency,  $Q$ , and temperature coefficient with changes in the relative thickness and position of the ceramic disk and has been applied to some new filter types at 455 kHz.

One of the most important characteristics of piezoelectric transducers is the capability of operating in various modes of vibration, as well as use in a large number of mechanical configurations.

A good example is shown in Fig. 8 where flexure modes

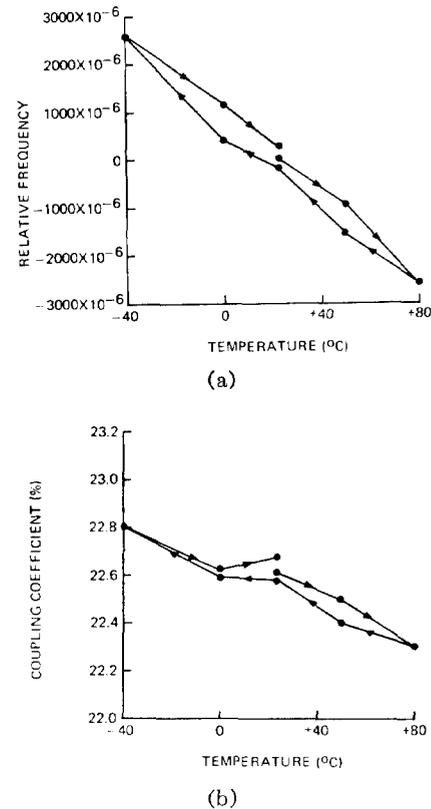


Fig. 26. Characteristics of PZT-type piezoelectric transducers (Murata). (a) Frequency variation. (b) Coupling coefficient variation with change of temperature.

are excited by composite ceramic/metal alloy transducers. In addition, the high electromechanical coupling coefficient of the ceramic makes it possible to design very stable wide-bandwidth (10–20 percent) low-frequency filters of this type.

A great deal of analytical work has been accomplished in Japan in the area of describing the electromechanical characteristics of composite bending transducer resonators. Konno's and Kusakabe's published work on this particular subject alone totals more than 200 pages and includes twisted bars, mass loaded bars, and bars that vibrate in nondegenerate modes, as well as simple bars driven with ceramic plates that only partially cover the upper and lower surfaces [59]. A very detailed analysis of a fully covered bar has been achieved by Okamoto *et al.* [60], and is summarized in [19].

#### LOOKING TO THE FUTURE

We will look at the next few years based on the direction of our present technology. There will be breakthroughs of course such as the monolithic filter of some years ago, but these are difficult to predict, even if one is playing an active part in the development. Some of the more predictable areas are size reduction, an expanded use of low-frequency filters, an improvement of delay and ripple characteristics, the realization of multiresonator monolithic filters with finite attenuation poles, as well as various improvements in the discrete element

mechanical filter as a result of the large amount of effort being expended on monolithic filters.

### Size Reduction

In terms of mechanical filters, size reduction is still a relatively unexplored technology. In the case of disk-wire mechanical filters a size reduction from 5.0–1.3 cm<sup>3</sup> was made with relatively little effort and no breakthroughs in technology other than the use of a lower order mode of vibration. By making use of ceramic transducers and small diameter disks the volume can be reduced by 2:1 without a great deal of difficulty. At the present time, the longitudinal-mode bar, flexural-coupling rod design independently developed by Börner [54] and Konno [18], (see Fig. 27) is packaged in less than 1 cm<sup>3</sup>. Although it may be somewhat more difficult (in comparison with the disk-wire filter) to reduce the size of this design because of the fixed length of the resonators, it is possible to do so by decreasing the diameter of the rods, but only to the point where flexural modes become a problem. Discrete element filters such as the disk or rod types actually have an advantage over monolithic structures (Fig. 28) in that relatively large plating surfaces are not needed to reduce both the filter impedance level and the effects of stray capacity. Disk and rod-type filters are low-impedance devices because of the high dielectric constant of PZT transducer materials, or in the case of magnetostrictive ferrites, the use of low-turn coils. In addition, only two transducers are needed, resonator spacing is commonly 0.025 cm regardless of filter bandwidth, and none of the volume is used for the purpose of reducing reflections from a boundary and very little is used for support.

It must be said that there are also some basic limitations as to how small we can build a discrete component mechanical filter. One, of course, is that of fabricating the parts and assembling them without excessive frequency shift and miscoupling. A second limitation, as we discussed earlier, is that of spurious modes due to having to fix one of the dimensions thus decreasing the frequency spacing of nearby bending modes [54]. Another size-reduction limitation relates to the nonlinearity of the transducer and resonator materials as a function of size. Yakuwa has made a very interesting comparison of ferrite core inductors, piezoelectric ceramics, nickel alloys, and quartz, showing the relationship between various loss factors and minimum size for a specific bandwidth filter [61]. A final consideration is the three-dimensional shape of most discrete element mechanical filters, which not only prevents the use of planar manufacturing processes but ultimately may be the most serious volume-reduction limitation.

### Low Frequencies

The large amount of development work on low-frequency filters in Japan will most probably be felt in other parts of the world. Fig. 29 shows, for example,

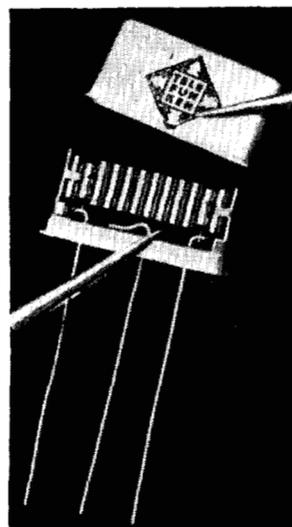


Fig. 27. Miniature 455-kHz rod-wire mechanical filter (1 cm<sup>3</sup>).

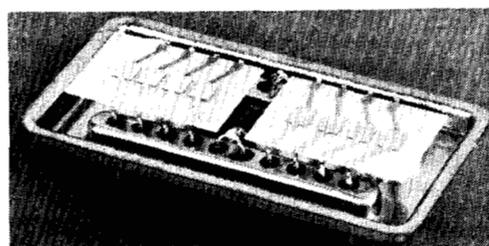


Fig. 28. Eight-pole monolithic quartz mechanical filter.

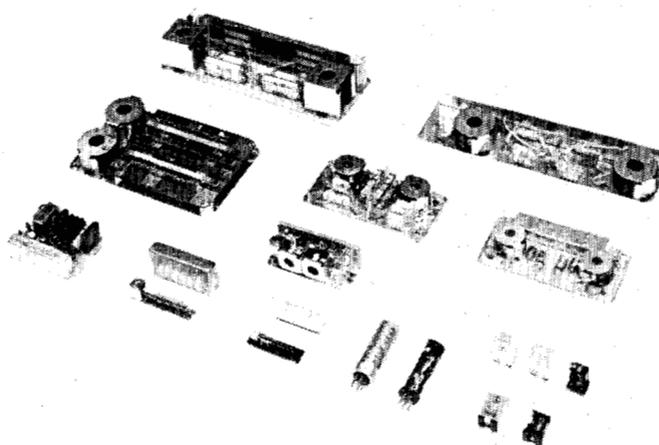


Fig. 29. Low-frequency mechanical filters (Fujitsu).

some of the wide variety of filters being manufactured by Fujitsu Ltd. Most applications of low-frequency mechanical filters involve selecting single tones, which in turn means that only one or two resonators are needed. Therefore, the selectivity is dependent solely on resonators that are coupled to the external mounting structure. This results in lower resonator  $Q$  and microphonic responses due to external vibrations.

The microphonics problem has been solved to some degree by the use of highly damped supports. This has made it possible to use fundamental modes of vibration,



It has already been shown that thickness modes can be trapped in much the same way as the shearing modes [70], which leads us to suspect that other new configurations will be developed in the next few years.

A large amount of technology has been devoted to the monolithic mechanical filter manufacturing process, for instance in areas such as the use of a laser to vary the coupling between electrode pairs and the fabrication of extremely flat and parallel crystal blanks. It is certain that this work will continue for a long period of time and should be applicable to mechanical filter technology in general.

#### *Developments in Other Countries*

Mechanical filter development outside of Japan, West Germany, and the U. S. has primarily been concentrated on distributed line filters of the type shown in Fig. 7(a). This has been the case in Great Britain [71], Poland [72], the USSR [73], and East Germany, where they have been manufactured for some years. Recent papers published in England have reflected an interest in transducer design [74] and low-frequency flexure mode resonators [75]. A study of flexure modes in mechanical filters is also a subject of interest in Czechoslovakia [76]. At this time, channel filters of the configuration of Fig. 5(b) are manufactured in East Germany (200 kHz) [77] and in Czechoslovakia (64–108 kHz) [78].

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